Exercise 25

Differentiate.

$$f(x) = \frac{x}{x + \frac{c}{x}}$$

Solution

Use the quotient rule to differentiate $f(x)$.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\frac{x}{x + cx^{-1}}\right) \\ &= \frac{\left[\frac{d}{dx}(x)\right](x + cx^{-1}) - \left[\frac{d}{dx}(x + cx^{-1})\right](x)}{(x + cx^{-1})^2} \\ &= \frac{(1)(x + cx^{-1}) - (1 - cx^{-2})(x)}{(x + cx^{-1})^2} \\ &= \frac{2cx^{-1}}{(x + cx^{-1})^2} \\ &= \frac{\frac{2c}{x}}{\left(x + \frac{c}{x}\right)^2} \end{aligned}$$